Gauge Coupling Unification with Extra Dimensions and Correction due to Higher Dimensional Operators

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Abstract

We study the gauge coupling unification with extra dimensions. We take into account corrections due to the higher dimensional operators. We show the prediction of $\alpha_3(M_Z)$ is sensitive to such corrections, even if $c < \Phi > /M = O(0.01)$. We also discuss the $b - \tau$ Yukawa unification.

Recently, theories with large extra dimensions have been studied intensively [1]-[5]. If such extra dimensions correspond to a TeV scale that can be a solution of the naturalness problem. One of interesting aspects in the theory with extra dimensions is the power-law behaviour of the running gauge coupling constants shown in Ref. [6], that is, the towers of Kaluza-Klein excitation modes lead to the power-law behaviour. That provides with the possibility that the three gauge coupling constants of the standard model are unified at lower energy scale than 10^{16} GeV and the unified energy scale is just above the energy scale μ_0 where the Kaluza-Klein excitation modes appear.

However, detailed analyses on the minimal matter content and canonical level of $U(1)_Y$ show that the predicted value of $\alpha_3(M_Z)$ increases as μ_0 decreases from 10^{16} GeV to a TeV scale, and we obtain incorrect prediction for $\alpha_3(M_Z)$ [7, 8]. There are several works to obtain a realistic prediction of $\alpha_3(M_Z)$ leading to the experimental value [9],

$$\alpha_3(M_Z) = 0.119 \pm 0.002, \tag{1}$$

e.g. by considering the non-canonical level of $U(1)_Y$ different from 5/3 or adding extra matter fields [10, 8] [1].

In this paper we consider the case with the minimal matter content and the canonical level of $U(1)_Y$ equal to 5/3, and we take into account the correction due to the higher dimensional operator, e.g. $c(\Phi/M)FF$, where c is a coupling constant and M is the cut-off energy scale. We consider the $SU(5)$ grand unified theory (GUT) with the **24** Higgs field Φ as our framework. Within this framework, we have the correction term to the gauge kinetic term [12] [2],

$$-\frac{1}{4}c\frac{<\Phi_{\alpha\beta}>}{M}F^\alpha_{\mu\nu}F^{\beta\mu\nu}. \tag{2}$$

Note that the vacuum expectation value

$$<\Phi_{\alpha\beta}>\propto \text{diag}(1/2\sqrt{15})(2,2,2,-3,-3), \tag{3}$$

which corresponds to the $SU(3) \times SU(2) \times U(1)_Y$ preserving direction, contributes to the

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[1]See also Ref. [11].

[2]In Ref. [13], corrections due to Higgs fields with larger representations are discussed within the framework of the four-dimensional supersymmetric $SU(5)$ GUTs.

gauge coupling constants non-universally. Thus, the initial condition changes into

$$\alpha_i^{-1} = \alpha_X^{-1}(1 + C_i), \tag{4}$$

$$(C_1, C_2, C_3) = \frac{x}{2\sqrt{15}}(-1, -3, 2) \tag{5}$$

where $x = c < \Phi > /M$. With this initial condition, let us study the gauge coupling unification with extra dimensions, that is, the prediction of $\alpha_3(M_Z)$. We will also study the $b - \tau$ Yukawa unification.

First, we give the set-up of our model and its renormalization group (RG) equations [6]. Following ref. [6] we assume that only the gauge boson and Higgs supermultiplets of the minimal supersymmetric standard model (MSSM) are in the bulk and have the towers of Kaluza-Klein states and that the lepton and quark supermultiplets are sitting at a fixed point of an orbifold on which the δ dimensional internal space is compactified so that they have no towers of Kaluza-Klein states. It is easy to extend to the case that some quarks or leptons have the Kaluza-Klein towers. Under these assumptions, the one-loop β-functions of the gauge couplings g_i $(i = 1, 2, 3)$ and the Yukawa couplings $g_{t,b,\tau}$ above μ_0 become [6] [3]:

$$(16\pi^2)\beta_1 = g_1^3 \left(6 + \frac{6}{5}(Y_\delta/2)(\frac{\Lambda}{\mu_0})^\delta\right), \tag{6}$$

$$(16\pi^2)\beta_2 = g_2^3 \left(4 - 6(Y_\delta/2)(\frac{\Lambda}{\mu_0})^\delta\right), \tag{7}$$

$$(16\pi^2)\beta_3 = g_3^3 \left(3 - 12(Y_\delta/2)(\frac{\Lambda}{\mu_0})^\delta\right), \tag{8}$$

$$(16\pi^2)\beta_t = g_t \left[3g_t^2 - \frac{3}{10}g_1^2 - \frac{3}{2}g_2^2 \right.$$
$$\left. +(Y_\delta/2)(\frac{\Lambda}{\mu_0})^\delta(6g_t^2 + 2g_b^2 - \frac{17}{15}g_1^2 - 3g_2^2 - \frac{32}{3}g_3^2)\right], \tag{9}$$

$$(16\pi^2)\beta_b = g_b \left[3g_b^2 + g_\tau^2 - \frac{3}{10}g_1^2 - \frac{3}{2}g_2^2 \right.$$
$$\left. +(Y_\delta/2)(\frac{\Lambda}{\mu_0})^\delta(2g_t^2 + 6g_b^2 - \frac{1}{3}g_1^2 - 3g_2^2 - \frac{32}{3}g_3^2)\right], \tag{10}$$

$$(16\pi^2)\beta_\tau = g_\tau \left[3g_b^2 + g_\tau^2 - \frac{3}{10}g_1^2 - \frac{3}{2}g_2^2 \right.$$
$$\left. +(Y_\delta/2)(\frac{\Lambda}{\mu_0})^\delta(6g_\tau^2 - 3g_1^2 - 3g_2^2)\right], \tag{11}$$

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[3]In Ref.[8] β-functions of soft supersymmetry breaking parameters have also been obtained by use of the recently developed technique based on the spurion formalism [14, 15].

where we have neglected the Yukawa couplings of the first and second generations, and Y_δ is defined as [8]

$$Y_\delta = \frac{\pi^{\delta/2}}{\Gamma(2 + \delta/2)} \ . \tag{12}$$

This coefficient corresponds to X_δ in Ref. [6],

$$X_\delta = \frac{\pi^{\delta/2}}{\Gamma(1 + \delta/2)} \ , \tag{13}$$

but these are different each other by the factor $1 + \delta/2$. In contrast with Ref. [6], in Ref. [8] the matching condition between the four-dimensional effective theory and $D + \delta$ dimensional theory is required to obtain Y_δ such that the evolution equations of the couplings in the effective theory smoothly go over in the large compactification radius limit to those in the uncompactified, original, $D + \delta$ dimensional theory. In particular, the continuous Wilson RG approach, which is applicable in any dimensions, is employed.

Below the energy scale μ_0, we use the two-loop RG equations of the four dimensional MSSM. For simplicity, we take $\delta = 1$. Under the initial condition (5), we predict $\alpha_3(M_Z)$ using these RG equations with the experimental values [9],

$$M_\tau = 1.777 \text{ GeV}, \quad M_Z = 91.188 \text{ GeV}, \tag{14}$$

$$\alpha_{\text{EM}}^{-1}(M_Z) = 127.9 + \frac{8}{9\pi} \log \frac{M_t}{M_Z} \ , \tag{15}$$

$$\sin^2 \theta_W(M_Z) = 0.2319 - 3.03 \times 10^{-5} T - 8.4 \times 10^{-8} T^2 \ , \tag{16}$$

where $T = M_t/[\text{GeV}] - 165$. Here M_τ and M_t are the physical tau and top quark masses, where we take $M_t = 174.1$ GeV in our analyses.

The prediction of $\alpha_3(M_Z)$ is shown in Fig.1. The four lines correspond to $x = 0.00, 0.01, 0.03$ and 0.05. The uppermost line corresponds to $x = 0.00$, while the lowest line corresponds to $x = 0.05$, that is, the predicted value $\alpha_3(M_Z)$ decreases as x increases. For example, for $x = 0.05$ we find a good agreement of $\alpha_3(M_Z)$ with the experimental value at $\mu_0 = 10$ TeV, while we obtain a good prediction for $x = 0.03$ at $\mu_0 = 10^{10}$ GeV. Thus, non-vanishing values of x can lead to the precise value $\alpha_3(M_Z)$ even for $\mu_0 \neq 10^{16}$ GeV. The suitable value of μ_0 is very sensitive to x, and it changes from a TeV scale to 10^{16} GeV when we vary x by $O(0.05)$. Negative values of x lead incorrect $\alpha_3(M_Z)$. Similarly, we can calculate the case with $\delta > 1$. For larger δ, we find larger $\alpha_3(M_Z)$ as shown in Ref.[7].
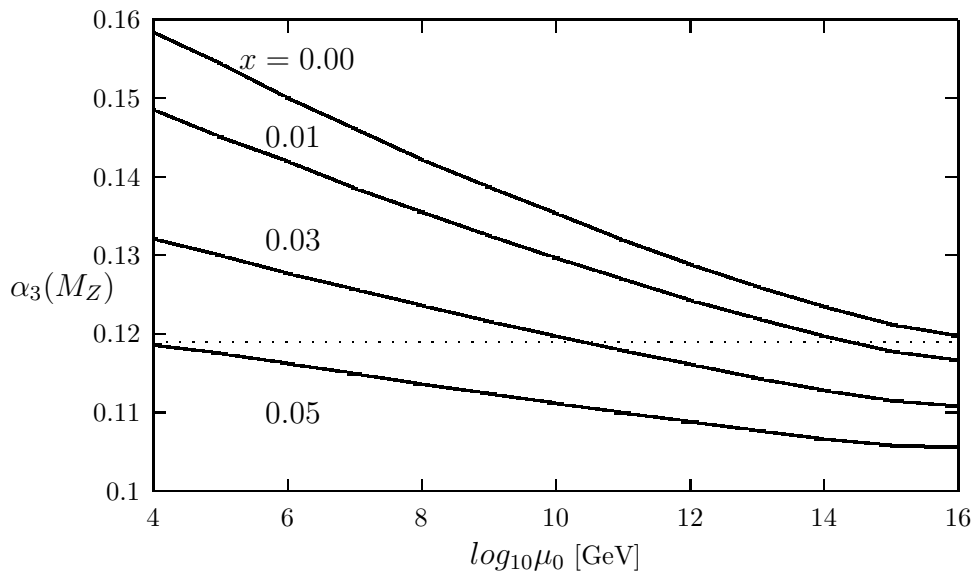
Fig.1: The prediction of $\alpha_3(M_Z)$

Change of the running behaviour of g_i $(i = 1, 2, 3)$ due to x affects the running behaviour of other couplings, e.g. the running behaviour of the Yukawa couplings. Furthermore, the Yukawa couplings have corrections due to higher dimensional operators. The bottom and tau Yukawa couplings have the correction term, $c'(\Phi/M)(\mathbf{10})(\bar{\mathbf{5}})H_d$, and that changes the initial condition of the $b - \tau$ Yukawa unification, $g_b(1 + C_b) = g_\tau(1 + C_\tau)$ with the ratio $C_b/C_\tau = -2/3$. Naturally, C_b and C_τ would be of the same order as those of the gauge coupling corrections C_i, but there is no closer relation between C_b (C_τ) and C_i, because c and c' couplings are independent of each other [4].

Now let us calculate effects of x and C_b (C_τ) on the bottom mass $m_b(M_Z)$ under the $b-\tau$ Yukawa unification in the theory with an extra dimension. The results for $C_b = C_\tau = 0$ are shown in Fig. 2. We have taken $\tan\beta = 3$. The three lines in Fig.2 correspond to $x = 0.00$, 0.03 and 0.05. The uppermost (lowest) is for $x = 0.00$ (0.05). As x increases, the bottom mass $m_b(M_Z)$ decreases for any value of μ_0. The case with $\mu_0 = 10$ TeV and $x = 0.05$ leads to slightly smaller bottom mass than a combination of higher μ_0 and smaller $|x|$ leading a good prediction of $\alpha_3(M_Z)$, e.g. $(x, \mu_0[\text{GeV}]) = (0.03, 10^{10})$ or $(0.00, 10^{16})$.

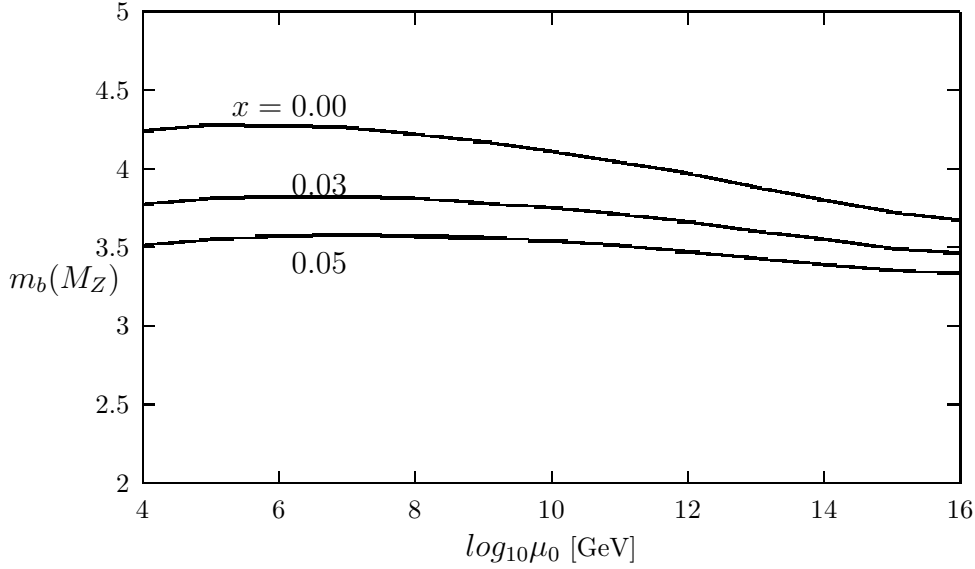[4]Within the framework of gauge-Yukawa unified theories, these couplings could be related each other [16].

Fig.2:The bottom mass

Similarily, we can discuss the case with non-vanishing C_b and C_τ. As an example, let us consider the initial condition that g_b is less than g_τ by 3%. In this case, we have $m_b(M_Z) = 3.40$ GeV for $\mu_0 = 10$ TeV and $x = 0.05$ and $m_b(M_Z) = 3.65$ GeV for $\mu_0 = 10^{10}$ GeV and $x = 0.03$. These values are different from the corresponding values in Fig.2 by 3%. Thus, if we introduce a few percentage of the difference for the $b-\tau$ Yukawa unification, the predicted values of $m_b(M_Z)$ are shifted from values in Fig.2 by almost same percentage.

Similarily, we can discuss the case with large $\tan\beta$. Indeed, the case with $\tan\beta = 20$ has a behaviour very similar to Fig. 2. Furthermore, much larger $\tan\beta$ leads to smaller $m_b(M_Z)$ as shown in Ref. [8]. However, in the large $\tan\beta$ we have sizable SUSY corrections to the bottom mass [17, 18]. They depend on superparticle masses and increase as $\tan\beta$ incerases. For example, the case with $\tan\beta = 20$ has $O(10\%)$ of SUSY corrections to the bottom mass. Thus, the detailed prediction of the bottom mass for large $\tan\beta$ is beyond our scope.

To summarize, we have studied the effects due to the higher dimensional operators on the gauge coupling unification as well as the running behaviour of the Yukawa couplings. The energy scale μ_0 leading to a good prediction of $\alpha_3(M_Z)$ is very sensitive to x even for $x = O(0.01)$. For example, in the case with $x = 0.05$ the gauge coupling unification can be realized for $\mu_0 = O(10)$ TeV.

Note added

After completion of this work, an article [19] appeared, where effects due to higher dimensional operators to the gauge couplings are also discussed.

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